Hyperbolic Heat-Conduction Problems: Numerical Simulations via Explicit Lax-Wendroff-Based Finite Element Formulations

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The paper describes numerical simulations for hyperbolic heat-conduction problems involving non-Fourier effects via explicit self-starting Lax-Wendroff-based finite element formulations. For cases involving extremely short transient durations or for very low temperatures near absolute zero, the classical Fourier diffusion model for heat conduction breaks down since the wave nature of thermal energy transport becomes dominant. Major difficulties in numerical simulations include severe oscillatory solution behavior in the vicinity of the propagating shocks. The present paper describes an alternate methodology and different computational perspectives for effective modeling/analysis of hyperbolic heat-conduction models involving non-Fourier effects. In conjunction with the proposed formulations, smoothing techniques are incorporated to stabilize the oscillatory solution behavior and to accurately predict the propagating thermal disturbances. The capability of exactly capturing the propagating thermal disturbances at characteristic time-step values is noteworthy. Numerical test cases are presented to validate the proposed concepts for hyperbolic heat-conduction problems.

Nomenclature

C	= see Eq. (25b)
c_T	= speed of thermal energy transport
c_v	= specific heat
E	= flux
F	= thermal load vectors
H	= load term, see conservation form, Eq. (16)
k	= thermal conductivity
M	= see Eq. (25a)
N_i	= thermal interpolation functions
T	= temperature
t	= time
U	= see conservation form, Eq. (16)
α	= thermal diffusivity
$\Delta\xi$	= time step
γ	= algorithm stability parameter, see Eq. (26)
η	= nondimensional quantity
ρ	= density
θ	= nondimensional quantity
ξ	= nondimensional quantity
τ	= relaxation parameter due to non-Fourier effect
Ω	= domain
Γ	= boundary

Subscripts

t	= time derivative

Superscripts

n, $n + ½$, $n + 1$	= time levels

Introduction

THE phenomenon of hyperbolic heat conduction involves thermal energy transport that accounts for finite speeds of propagation as opposed to an infinite speed of thermal energy transport. For most engineering modeling/analysis relevant to heat transfer in structures and materials, the notion of infinite speeds of thermal energy transport seems acceptable. However, several pathological anomalies exist for the classical Fourier heat-conduction models, especially for cases involving extremely short transient durations or for very low temperatures near absolute zero. As a consequence, the mode of heat conduction is no longer diffusive or parabolic, but is propagative or hyperbolic. Although the convergence time span between the hyperbolic and parabolic models is quite small, it may become important when extremely short times are involved.

The concept of the "hyperbolic nature" of the heat-conduction equation dates as far back as Maxwell[1] and evidence of heat-transport velocity of approximately 10^8 cm/s has been observed by Brorson et al.[2] upon heating thin gold films with laser pulses. Related experimental data appear concerning the hyperbolic nature of heat conduction.[2–7] Various different approaches[8–17] have since been derived concerning the hyperbolic nature of the heat-conduction equation. Most of these approaches are based on the general notion of relaxing the heat flux, thereby introducing a non-Fourier effect. Some analytical solutions for the hyperbolic heat-conduction equation appear in Refs. 18–22. Numerical simulations using finite-difference approximations for hyperbolic heat-conduction models influenced by various boundary conditions and the effects of variable thermal parameters appear due to Özisik et al.[23,24] and Glass et al.[25–28] In the context of new solution techniques and numerical simulations, that by Carey and Tsai[29] and recently by Tamma et al.[30–32] involve finite elements for modeling/analysis of hyperbolic heat-conduction problems involving non-Fourier effects. Carey and Tsai[29] investigate the effectiveness of numerical solution techniques for the case of propagating thermal disturbances, and they report the use of backward-difference integration as being helpful for suppressing the numerical oscillations in the solution, whereas Tamma et al.[30–32] employ specially tailored hybrid transfinite element formulations for accurately capturing the propagating thermal disturbances. Nonetheless, most often the major difficulties in the analytical developments include complex mathematical formulations for obtaining closed-form analytical solutions, and in the numerical simulations, the difficulties include severe oscillations when sharp fronts and reflective boundaries are involved and handling (capturing) the sharp discontinuities at the wave front with high resolution.

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The present paper describes explicit finite element formulations for modeling/analysis of hyperbolic heat-conduction problems, and it provides an effective computational methodology with features incorporating smoothing concepts for predicting the representative transient behavior of the propagating thermal disturbances. Constant thermophysical properties are assumed, although general nonlinear effects that normally occur in thermal heat transfer may be readily incorporated. Representative sample numerical test models are described to validate the proposed developments.

Fourier Versus Non-Fourier Hyperbolic Heat-Conduction Model

Fourier's model in the classical theory of heat conduction assumes

$$q = -k\nabla T \tag{1}$$

where q is the heat flux postulated to be directly proportional to the temperature gradient. As a consequence, the classical heat-conduction equation derived from the continuity equation can be represented (in the absence of a heat source) as

$$\rho c_v \dot{T} + \nabla q = 0 \tag{2}$$

in the domain Ω and is parabolic in nature. Such a notion of instantaneous heat diffusion does yield fairly accurate temperature predictions for most commonly encountered practical situations, although several pathological anomalies exist, especially for cases involving extremely short transient durations or for very low temperatures near absolute zero. This, in particular, is due to the fact that the thermal energy transport no longer travels with an infinite speed, but only with a finite speed of propagation. In order to account for the finite nature of heat propagation and to eliminate the paradox of infinite speed of thermal energy transport, the notion of a time-dependent relaxation model based on the idea of relaxing the heat flux in the Fourier model for heat conduction leads to

$$q + \tau\frac{\partial q}{\partial t} = -k\nabla T \tag{3}$$

where τ is a relaxation time parameter. The governing heat-conduction equation involving non-Fourier effects can now be obtained by introducing the modified Fourier model, Eq. (3), into the energy conservation equation, Eq. (2), which leads to (Fig. 1)

$$\frac{1}{c_T^2}\ddot{T} + \frac{1}{\alpha}\dot{T} = \nabla^2 T \tag{4}$$

and is hyperbolic in nature. In Eq. (4), c_T is the speed of thermal heat transport and is given by

$$c_T = (k/\tau\rho c_v)^{1/2} = (\alpha/\tau)^{1/2} \tag{5}$$

and $\alpha = k/\rho c_v$ is the thermal diffusivity.

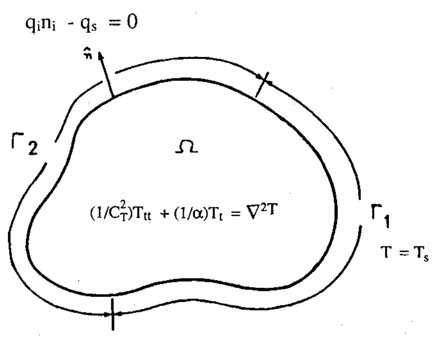

Fig. 1 Description of hyperbolic heat-conduction thermal problem.

It is evident from the above that as $c_T \to \infty$ (i.e., the limiting case of zero relaxation time), then $\tau \to 0$, which is the case of infinite speed of heat propagation, and that Eq. (4) reverts back to the classical parabolic heat-conduction model given by Eq. (2) where Fourier's law applies, since Eq. (3) readily reduces to the classical Fourier's model given by Eq. (1). Furthermore, at steady state, the non-Fourier model reverts to Fourier's model although the relaxation parameter $\tau \neq 0$, and, as a consequence, the temperature solutions differ for the two models only during the transient state. The relaxation parameter τ physically signifies the initiation of heat flow after a temperature gradient has been imposed. Consequently, the initiation or cease of heat flow does not occur instantaneously but rather occurs gradually after application or removal of the temperature gradient, respectively.

Typical boundary conditions may be represented as follows. On Γ_1:

$$T = T_s \tag{6a}$$

On Γ_2:

$$q_i n_i = q_s \tag{6b}$$

where $\Gamma = \Gamma_1 + \Gamma_2$. The first boundary condition is the specification of prescribed temperature on Γ_1, and the second boundary condition is associated with surface heat flux q_s on Γ_2. The initial conditions are given by

$$T(x_i, t=0) = T_i, \qquad \dot{T}(x_i, t=0) = \dot{T}_i \tag{7}$$

For the model test problems presented in this paper, we consider the hyperbolic heat transport in domain $\Omega = (0, \ell)$ subject to quiescent initial conditions given as

$$T(x, 0) = 0, \qquad \dot{T}(x, 0) = 0 \tag{8}$$

The following types of boundary conditions are considered. Model 1:

$$T(0, t) = T_s, \qquad T(\ell, t) = 0 \tag{9}$$

Model 2:

$$T(0, t) = T_s, \qquad \frac{\partial T}{\partial x}(\ell, t) = 0 \tag{10}$$

In the preceding models, the thermal disturbance propagates from the left boundary $x = 0$ at $t = 0^+$ and encounters the right boundary after time $t = \ell/c_T$.

Nondimensional Equations

For convenience, representative nondimensionalized equations are used. Introducing the dimensionless variables

$$\theta = T/T_s, \qquad \xi = c_T^2 t/2\alpha, \qquad \eta = c_T x/2\alpha \tag{11}$$

leads to the nondimensional representation of the governing equations for the hyperbolic heat-conduction models as

$$\frac{\partial^2\theta}{\partial\xi^2} + 2\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} \tag{12}$$

with boundary conditions as follows.

Model 1:

$$\theta(0, \xi) = 1, \qquad \theta(\ell, \xi) = 0 \tag{13}$$

Model 2:

$$\theta(0, \xi) = 1, \qquad \frac{\partial\theta}{\partial\eta}(\ell, \xi) = 0 \tag{14}$$

Initial conditions:

$$\theta(\eta, 0) = 0, \qquad \dot{\theta}(\eta, 0) = 0 \tag{15}$$

Numerical Computational Methodology: Explicit Architecture and Finite Element Representations

For the modeling/analysis of hyperbolic heat-conduction problems, we present a new and effective numerical computational methodology that may be regarded as a finite-element-based Lax-Wendroff-type approach. Tamma and Namburu[33,34] recently proposed explicit second-order-accurate finite element formulations for nonlinear/linear computational structural dynamics. More recently, alternate representations appear due to Tamma and Namburu[35-37] for the effective solution of nonlinear/linear computational structural dynamics and unified thermal-structural dynamic problems. The methodology described here for applicability to hyperbolic heat-conduction problems seeks to effectively model the physics and nature of the propagating thermal energy transport accurately, and efficiently and is based on recent developments.[36]

Applications employing finite elements for formulating Lax-Wendroff-type schemes for shock-wave calculations appear in Oden et al.,[38,39] where finite-difference approximation to the governing equations are expressed as conservation laws and temporal discretization is accomplished in the spirit of the Lax-Wendroff equations.[40,41] On analogous principles, the Taylor-Galerkin approach proposed by Donea et al.[42,43] for flow problems seeks to introduce more analytical information into the numerics, and in principle can be considered as a Lax-Wendroff-type formulation.[40,41] The methodology adopted in this paper for hyperbolic heat-conduction problems may be regarded as a finite-element-based Lax-Wendroff methodology of computation based on recent developments (see Tamma and Namburu[36]), which, besides providing better physical interpretation of the resulting discretized equations, also seeks to permit introduction of general boundary conditions in the most direct and natural manner in conjunction with several enhanced implementation characteristics. Although the numerical test models presented consider constant thermal properties, the formulations readily permit introduction of general nonlinear conditions and the proposed computational methodology is second-order-accurate, robust, and easy to implement. The ability to accurately capture the thermal disturbances at characteristic time-step values is especially noteworthy, and applicability of smoothing concepts for hyperbolic heat-conduction problems is new.

Computational Formulations

We first consider the representation of the governing nondimensionalized form of hyperbolic heat conduction in conservation form as

$$\frac{\partial U}{\partial \xi} + \frac{\partial E}{\partial \eta} = H \tag{16}$$

where the terms U, E, and H are given as

$$U = \frac{\partial \theta}{\partial \xi}, \qquad E = -\frac{\partial \theta}{\partial \eta}, \qquad H = -2U \tag{17}$$

Existing conventional time integration approaches for the discretization of wave propagation-type problems employ the form shown in Eq. (4), not the conservation form presented in Eq. (16), and first invoke the so-called "semidiscretization" process to yield a set of ordinary differential equations in time. Therein, finite-difference approximations are employed for the time derivatives for obtaining the necessary algorithmic relations that may be generally categorized as explicit methods and implicit methods. Unlike the aforementioned, the proposed formulations that are explicit in nature are first based on employing the conservation form of representation given by Eq. (16). In the spirit of the Lax-Wendroff-type formulations, the transient time-derivative term in conservation form is expressed in terms of a Taylor series expansion correct to second order (thus, the expression contains first- and second-order derivatives); these time derivatives are then evaluated from the governing transient equations expressed in conservation form as described subsequently. The resulting expression is now discretized in space employing the classical Galerkin formulations for deriving the explicit architecture and representations. Furthermore, the concept of flux representations are employed in conjunction with the aforementioned methodology to enhance the implementation characteristics and to permit natural introduction of general boundary conditions in the most direct and effective manner.

To advance the solution to the time level $n + 1$, following the spirit of the Lax-Wendroff-type formulations, we first consider the Taylor series expansion for U in time correct to second-order as follows:

$$U^{n+1} = U^n + \Delta\xi U_\xi^n + (\Delta\xi^2/2)U_{\xi\xi}^n + 0\ (\Delta\xi^3) \tag{18}$$

or

$$U^{n+1} = U^n + \Delta\xi U_\xi^{n+\frac{1}{2}} + 0\ (\Delta\xi^3) \tag{19}$$

where the subscript ξ indicates differentiation with respect to time. Substituting for $U_\xi^{n+\frac{1}{2}}$ in Eq. (19) directly from the governing transient hyperbolic (nondimensionalized) heat-conduction equation expressed in conservation form (Eq. 16), we have

$$U^{n+1} = U^n + \Delta\xi\ [H^{n+\frac{1}{2}} - E_x^{n+\frac{1}{2}}] \tag{20}$$

We now introduce the classical Galerkin formulations for deriving the discretized equations. As a result, after introducing the approximation for $H^{n+\frac{1}{2}}$ as

$$H^{n+\frac{1}{2}} = (H^n + H^{n+1})/2 \tag{21}$$

and the following spatial approximations

$$U = N_i U_i \tag{22}$$

$$E^{n+\frac{1}{2}} = N_i E_i^{n+\frac{1}{2}} \tag{23}$$

in conjunction with the classical Galerkin formulations, the discretized form of finite element representations are obtained in an explicit format as

$$[M + (\Delta\xi/2)C]\Delta U^{n+1} = F_c^n + F_1^{n+\frac{1}{2}} + F_2^{n+\frac{1}{2}} \tag{24}$$

where

$$M = \int_{\Omega^e} N_\alpha N_\beta\ d\Omega \tag{25a}$$

$$C = \int_{\Omega^e} N_\alpha N_\beta\ d\Omega \tag{25b}$$

$$F_c^n = -\Delta\xi C U^n \tag{25c}$$

$$F_1^{n+\frac{1}{2}} = \int_{\Omega^e} N_{\alpha,i} N_\beta\ d\Omega\ \{E^{n+\frac{1}{2}}\} \tag{25d}$$

$$F_2^{n+\frac{1}{2}} = -\Delta\xi \int_{\Gamma^e} N_\alpha N_\beta\ d\Gamma\ \{E^{n+\frac{1}{2}} \cdot \hat{n}\} \tag{25e}$$

The quantities on the right side of Eq. (24) are all known quantities being evaluated at time levels n and $(n + \frac{1}{2})$, respectively, where the $(n + \frac{1}{2})$ quantities can be directly evaluated employing $\theta^{n+\frac{1}{2}} = \theta^n + (\frac{1}{2})\Delta\xi\theta_\xi^n$. The quantity F_1 represents contribution from conduction heat transfer, and the vector

$\{E^{n+1/2}\}$ in Eq. (25d) represents element nodal heat fluxes due to conduction heat transfer. This vector can be readily evaluated by employing the classical conduction law, since temperatures are assumed to vary within each element. The quantity $\{E^{n+1/2} \cdot \hat{n}\}$ in Eq. (25e) represents element nodal heat fluxes normal to the element boundary surface, and, as a consequence, permits introduction of general nonlinear/linear natural boundary conditions in the most direct and effective manner without in any way disturbing the evaluation of the element integrals. The nature of the matrices appearing in Eq. (25) and the natural introduction of the boundary conditions without disturbing the element integrals significantly differs from those adopted via conventional finite element formulations for general wave propagation-type problems. For a truly explicit nature, the matrices M and C are diagonalized. As a consequence, the algorithm architecture permits evaluation of the system equations in an uncoupled manner without the need for solving simultaneous equations. Furthermore, for general nonlinear problems, the proposed formulations do not require iteration.

Since the proposed formulation directly yields U, which are rate quantities, we employ an updating scheme for evaluating θ as follows:

$$\theta^{n+1} = \theta^n + \Delta\xi \; [(1-\gamma)U^n + \gamma U^{n+1}] \tag{26}$$

where γ is a stability parameter introduced by Tamma and Namburu[36] and is selected so as to maintain the stability of the algorithm. The formulations presented are stable for $0.5 \le \gamma \le 1.0$, and the scheme is second-order-accurate for $\gamma = 1/2$. The stability and accuracy characteristics of the proposed formulations have been described elsewhere by Tamma and Namburu.[36]

Incorporation of Numerical Smoothing

When the proposed explicit finite element formulations are applied for the modeling/analysis of hyperbolic heat-conduction problems involving strong shocks, they are additionally stabilized by the incorporation of smoothing concepts. We demonstrate viable and effective smoothing features that lend themselves extremely well for these problems, especially, when sharp propagating fronts and reflective boundaries are involved.

The basic idea followed seeks to smooth or postprocess the solution U^{n+1} obtained from the explicit finite element formulations described in the previous section. As a consequence, we adopt the technique of Lapidus.[44] Used elsewhere by Lohner et al.[45] for high-speed compressible flows, the approach basically involves replacing the computed values of U^{n+1} by smoothed values U_s^{n+1} at the end of each time step $\Delta\xi$ calculated according to

$$U_s^{n+1} - U^{n+1} = -\Delta\xi \frac{\partial}{\partial\eta} \left| k_\eta \frac{\partial U^{n+1}}{\partial\eta} \right| \tag{27}$$

where the so-called artificial viscosity k_η is given for the present study as

$$k_\eta = Lh^2 \left| \frac{\partial v}{\partial\eta} \right| \tag{28}$$

In Eq. (28), L is an adjustable Lapidus constant, h is the representative mesh size, and v is associated with the component of velocity of the conservation variable. The coefficient k_η serves as an indicator and is proportional to the gradient of the conservation variables. As a consequence, the smoothing is typically employed only in the vicinity of the shock fronts.

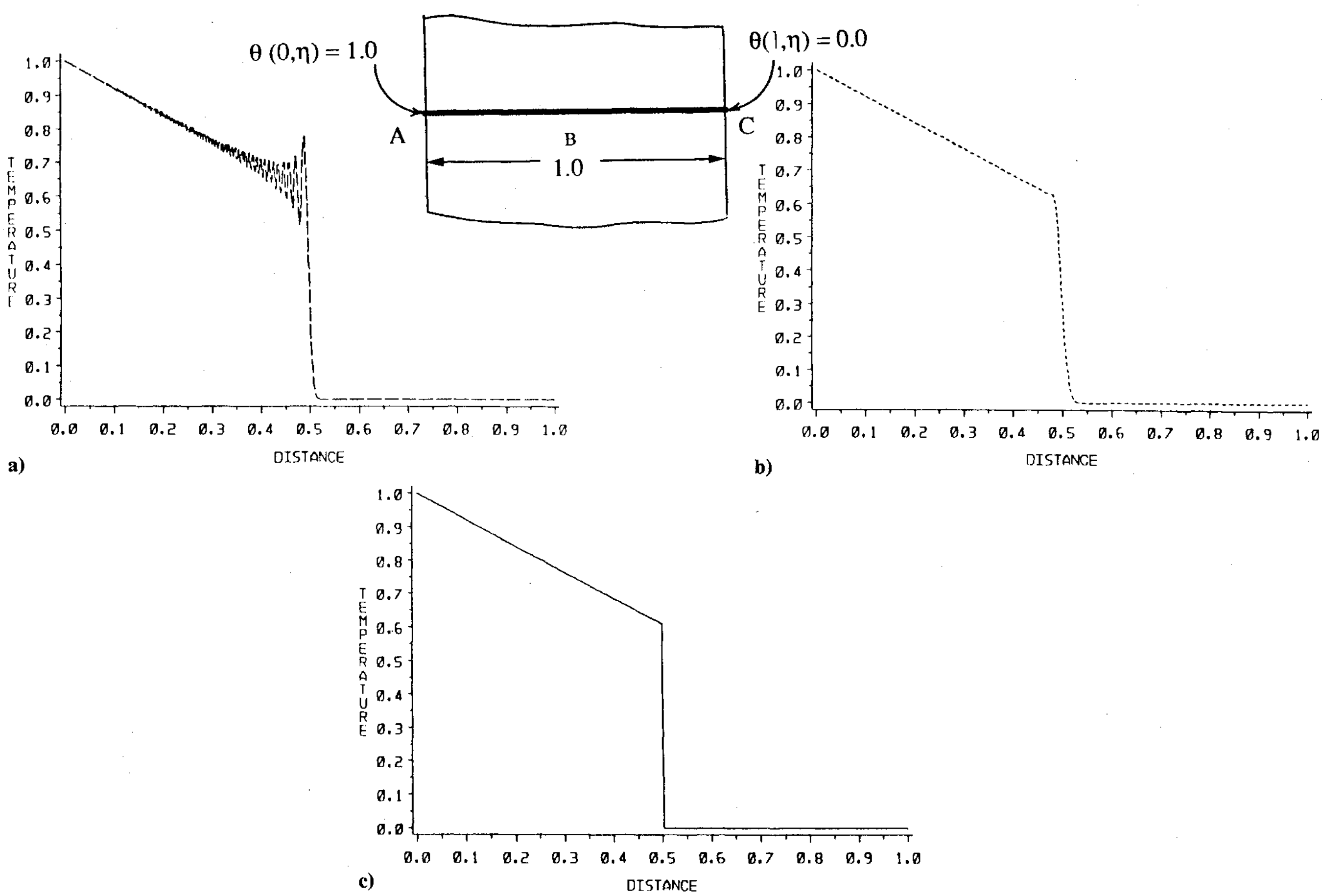

Fig. 2 Nondimensionalized temperature distributions in slab at $\xi = 0.5$ (model 1): a) solution response without smoothing; b) solution response introducing smoothing; and c) characteristic time-step solution response.

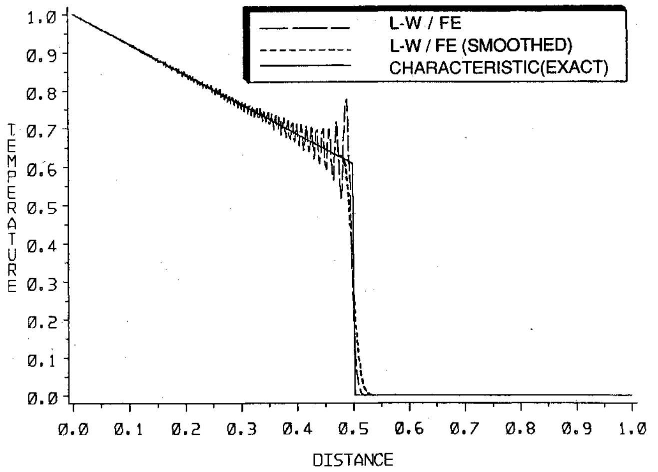

Fig. 3 Superimposed nondimensional temperature distributions in slab at $\xi = 0.5$ (model 1).

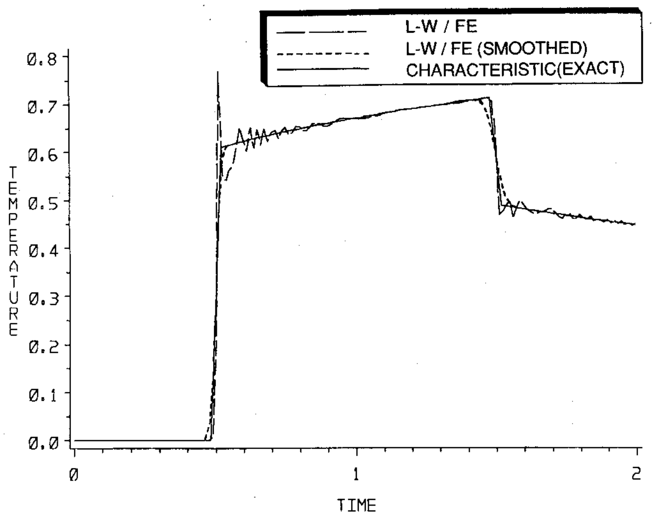

Fig. 4 Transient response of a point at center of slab (model 1).

Characteristic Time Step

The so-called characteristic time step is representative of the transit time required for a "wave" moving at unit speed to traverse one element. In this regard, it can be readily shown via the proposed explicit computational formulations that, at this value of the time step, the errors introduced by the finite element spatial discretization, the particular mass matrix, and temporal integration scheme all cancel to yield exact results. As a consequence, on employing $\Delta\xi = h$, which is the characteristic time step for the proposed formulations, the numerical results will yield exact nodal values no matter how few elements are employed. The characteristic time step also represents the stability limit or critical time step for the present explicit formulations, and, therefore, numerical computations cannot be performed at larger time-step values.

Illustrative Numerical Examples

The present explicit formulations that possess direct self-starting features are applied to sample hyperbolic heat-conduction models to demonstrate their ability to effectively predict the propagating thermal disturbances due to non-Fourier effects. Typical of computational schemes for similar problems, the present formulation exhibits an oscillatory solution behavior in the vicinity of the thermal wave front. Nonetheless, we additionally adopt features to smooth the numerical oscillations by introducing numerical smoothing as described previously. It is also noteworthy to mention that the present formulations yield an exact solution at the so-called characteristic time-step values mentioned earlier. At this value of the time step, the propagating thermal disturbance is accurately captured without exhibiting any oscillatory solution behavior. Illustrative numerical test models are presented next.

Model 1

The first test case concerns transient hyperbolic heat conduction in a one-dimensional slab in domain $\Omega(0, \ell)$. The thermal wave front propagates from the left boundary to the right boundary. The nondimensionalized boundary and initial conditions have been discussed earlier.

An exact analytic solution to this problem is available, and the problem was also attempted by Carey and Tsai.[29] The problem was modeled using a refined mesh of 500 two-noded linear elements, and a time step of $\Delta\xi = 0.00125$ was employed

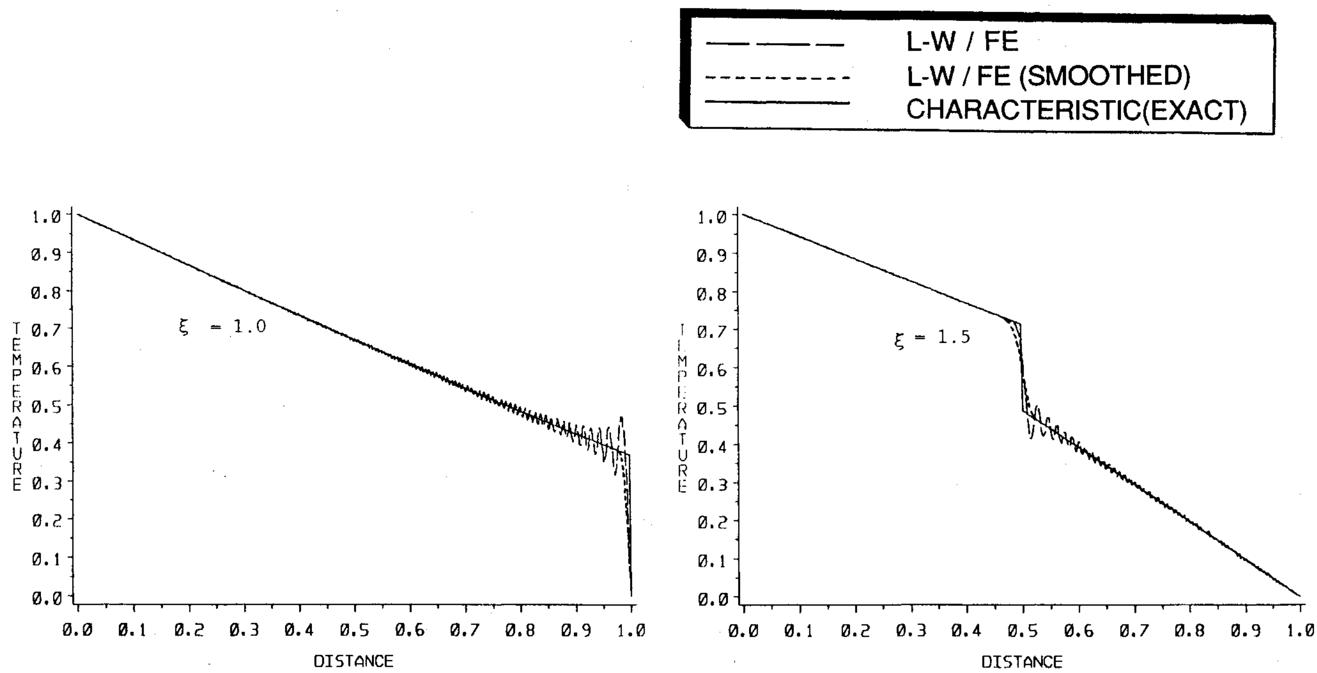

Fig. 5 Nondimensional temperature distributions in slab at $\xi = 1.0$ and 1.5, respectively (model 1).

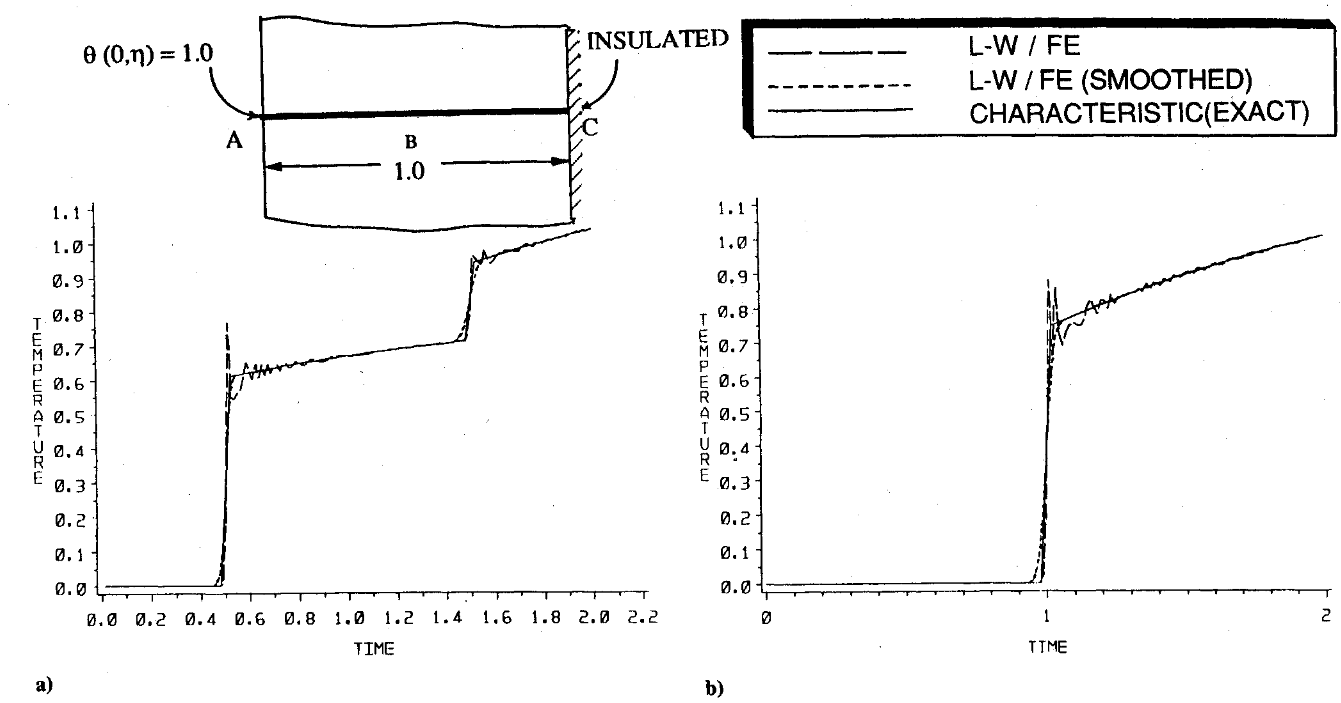

Fig. 6 Transient response of a point at center and right end of slab (model 2).

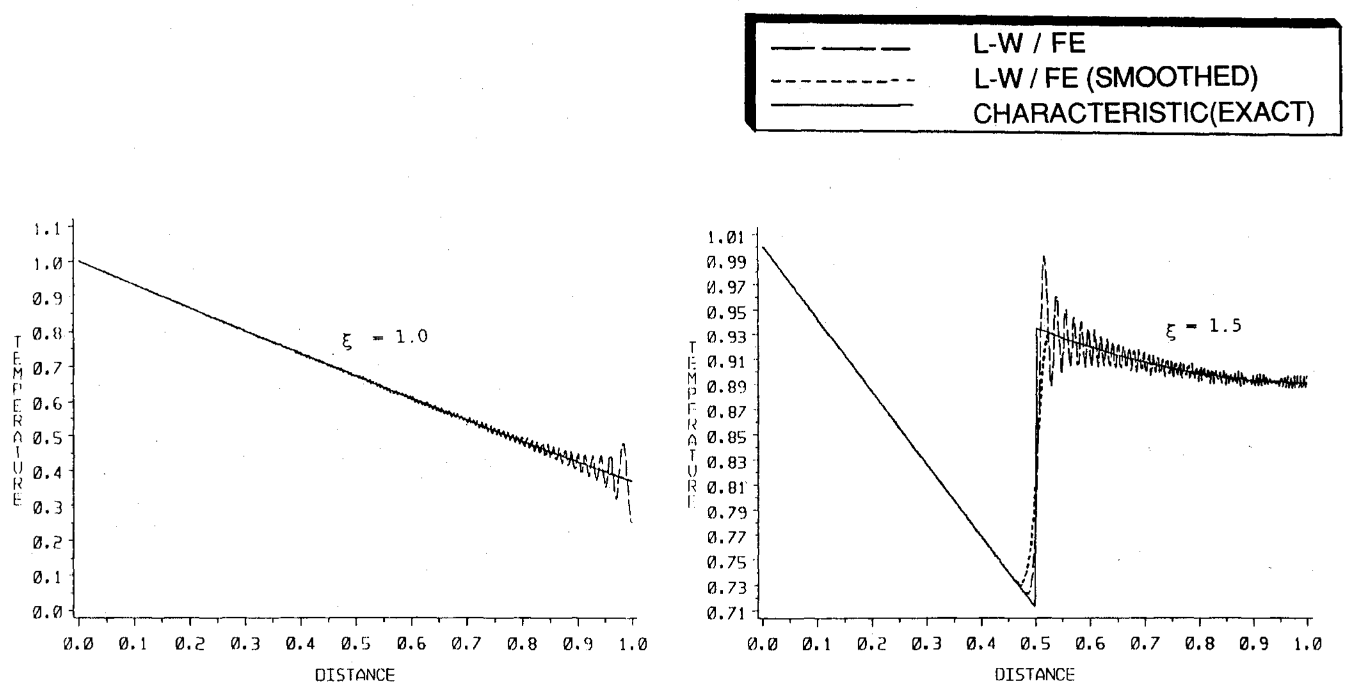

Fig. 7 Nondimensional temperature distributions in slab at $\xi = 1.0$ and 1.5, respectively (model 2).

in conjunction with the proposed formulations. In Fig. 2, the numerical simulations of the propagating thermal wave front are shown. In particular, nondimensionalized temperature distributions are plotted for model 1 at time $\xi = 0.5$. Figure 2a depicts the solution response obtained via the present formulation. The oscillatory solution behavior in the vicinity of the shock front is typical for most of the explicit time-integration schemes. Incorporating smoothing concepts, the oscillatory solution behavior is stabilized as shown in Fig. 2b. The characteristics of the solution are very sharp and clearly validate the capability to model this propagating thermal disturbance. In Fig. 2c, the solution response employing the characteristic time-step value is shown. At this value of the time step, the finite element solution obtained via the proposed formulation is exact and no smoothing is necessary. In Fig. 3, the superimposed temperature distributions with and without smoothing are shown in comparison to the exact solution. The transient thermal response of a point at the center of the slab is shown in Fig. 4. In Fig. 5, the temperature distributions at $\xi = 1.0$ and 1.5 are shown. The value of the time steps used were $\Delta\xi = 0.00125$ and 0.0015, respectively. The results obtained via the proposed formulation again clearly validate the capability of the model in the presence of sharp propagating fronts and reflective boundaries.

Model 2

The second test case concerns transient hyperbolic heat conduction in a one-dimensional slab in Ω (0, ℓ). Whereas in model 1 studied earlier the left boundary is maintained at $T = T_s$ and the right boundary is maintained at $T = 0$, in model 2, the left boundary is maintained at $T = T_s$ and the right boundary is assumed insulated.

An exact solution to this model is again available, and the problem was also attempted by Carey and Tsai.[29] This problem was modeled using a refined mesh of 500 two-noded linear elements, and a time step of $\Delta\xi = 0.00125$ was employed in conjunction with the proposed formulations. It is of interest here to evaluate the nature of the propagating thermal disturbances at $\xi = 0.5$, 1.0, and 1.5, respectively. The solution behavior for $\xi = 0.5$ is similar to the previous test case since the right boundary condition does not effect the response. However, in Fig. 6 the transient histories of a point at the center of the slab and the insulated boundary are shown, respectively. Although the double discontinuity shown in Fig. 6a at $\xi = 0.5$ (and $\xi = 1.5$) for a point at the center of the slab is due to the initial propagation and reflection of the thermal wave front, the same features will not be present for a point on the insulated boundary, where the discontinuity initially appears at $\xi = 1.0$, which is the time taken by the thermal disturbance to traverse the length of the slab. The temperature distributions at $\xi = 1.0$ and 1.5 are shown in Fig. 7, where $\Delta\xi = 0.001$ and 0.0015 were used as time-step values.

Concluding Remarks

The present paper described an explicit Lax-Wendroff-based finite element formulation for modeling/analysis of hyperbolic heat-conduction problems involving non-Fourier effects. The proposed formulations provide an alternate yet effective computational methodology for the modeling and analysis of the transient behavior of the propagating thermal wave front. Of the various added advantages, some of the distinguishing features include second-order accuracy, direct self-starting features given the initial conditions without the need to perform additional computations, capability to naturally apply general boundary conditions, and incorporation of smoothing concepts to stabilize the oscillatory behavior in the vicinity of the shock front. The capability of exactly predicting the propagating thermal disturbances using characteristic time-step values is noteworthy. Comparative results of numerical test models demonstrated excellent agreement, and the present methodology can be effectively used for the modeling/analysis of hyperbolic heat-conduction problems involving non-Fourier effects.

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